Exercícios propostos resolvidos

Capítulo 3 – Potenciação, Radiação e Produtos Notáveis

P 3.1

a) $a^3 \times a^{-3} = a^{3+(-3)} = a^0 = 1$

b) $(a^2)^{-3} = a^{-6} = \frac{1}{a^6}$

c) $(a \times b)^{-1} = \frac{1}{a.b}$

d) $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

P 3.2

a) $\left(\frac{11}{3}\right)^{-2} : \left(\frac{13}{3}\right)^{-1} = \left(\frac{3}{11}\right)^{2} : \left(\frac{3}{13}\right)^{1} = \frac{\cancel{9}}{121} \times \frac{13}{\cancel{3}} = \frac{3}{121} \times \frac{13}{1} = \frac{39}{121}$

b) $\left(\frac{1}{2} + 2\right)^{-3} : \left(1 + \frac{1}{4}\right)^{-2} = \left(\frac{1+4}{2}\right)^{-3} : \left(\frac{4+1}{4}\right)^{-2} = \left(\frac{5}{2}\right)^{-3} : \left(\frac{5}{4}\right)^{-2} = \left(\frac{2}{5}\right)^{3} : \left(\frac{4}{5}\right)^{2} = \frac{2^3}{5^3} \times \frac{5^2}{4^2}$

$= \frac{2^3}{5^3} \times \frac{5^2}{2^4} = \frac{1}{5^{3-2} \times 2^{4-3}} = \frac{1}{10}$

c) $\left(3 - \frac{1}{2}\right)^{-1} + \left(3 - \frac{1}{2}\right)^{-1} = \left(\frac{6-1}{2}\right)^{-1} + \left(\frac{6-1}{2}\right)^{-1} = \left(\frac{5}{2}\right)^{-1} + \left(\frac{5}{2}\right)^{-1} = \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

P 3.3

a) $0{,}01 = \frac{1}{100} = 10^{-2}$

b) $0{,}0001 = \frac{1}{10000} = 10^{-4}$

P 3.4

$$[(3a)^4:(5b)^2] \times \left[(3a)^{-1}(25b^3)\right] = \frac{3^4a^4}{5^2b^2} \times \frac{1}{3a} \times 5^6 \times b^3 = \frac{3^3a^3.b}{1} = 27a^3b$$

P 3.5

$$\left[(a.b)^3 \times \left(\frac{1}{a.b}\right)^{-2}\right] \div (a^5.b^5) = [a^3b^3 \times a^2b^2] \times \left(\frac{1}{a^5.b^5}\right) = [a^5b^5] \times \left(\frac{1}{a^5.b^5}\right) = 1$$

P 3.6

$$\left(\frac{3}{4}+0{,}5\right)^{-1}:\left(\frac{3}{4}-\frac{1}{2}\right)^{-2}=\left(\frac{3}{4}+\frac{1}{2}\right)^{-1}:\left(\frac{3-2}{4}\right)^{-2}=\left(\frac{3+2}{4}\right)^{-1}:\left(\frac{1}{4}\right)^{-2}=\frac{4}{5}:4^2$$
$$=\frac{4}{5}\times\frac{1}{4^2}=\frac{1}{20}$$

P 3.7

$$(1{,}2)^3+[(1-0{,}04)\times 3]:10^3=1{,}728+[(0{,}96)\times 3]:10^3=1{,}728+[2{,}88]:1000$$
$$=1{,}728+0{,}00288=1{,}73088$$

P 3.8

$$5\sqrt[3]{-27}+\ 2\sqrt[3]{27}=5\sqrt[3]{(-3)^3}+\ 2\sqrt[3]{3^3}=5\times(-3)+2\times 3=-15+6=-9$$

P 3.9

$$\sqrt{100-64}=\sqrt{36}=6$$

P 3.10

$$\sqrt{0{,}0016}=\sqrt{\frac{16}{10000}}=\frac{\sqrt{16}}{\sqrt{10^4}}=\frac{4}{10^2}=\frac{4}{100}=0{,}04$$

P 3.11

$$3\sqrt{50}-\ 2\sqrt{18}+\ \sqrt{98}=3\sqrt{25\times 2}-\ 2\sqrt{9\times 2}+\ \sqrt{49\times 2}=3\times 5\sqrt{2}-\ 2\times$$
$$3\sqrt{2}+\ 7\sqrt{2}=(15-6+7)\sqrt{2}=16\sqrt{2}$$

P 3.12

$$\sqrt{6}-\ \sqrt{24}+\ \sqrt{54}=\sqrt{6}-\ \sqrt{4\times 6}+\ \sqrt{9\times 6}=\sqrt{6}-\ 2\sqrt{6}+\ 3\sqrt{6}=$$
$$=(1-2+3)\sqrt{6}=2\sqrt{6}$$

P 3.13

$$2\sqrt{12}+\ \sqrt{27}-\ 3\sqrt{48}+\ \sqrt{108}=\ 2\sqrt{3\times 4}+\ \sqrt{3\times 9}-\ 3\sqrt{3\times 16}+\ \sqrt{3\times 36}=$$
$$2\times 2\sqrt{3}+\ 3\sqrt{3}-\ 3\times 4\sqrt{3}+\ 6\sqrt{3}=(4+3-12+6)\sqrt{3}=\sqrt{3}$$

P 3.14

$$\sqrt[3]{3\sqrt{2}}\,\sqrt[3]{\sqrt{3^2.2}} = \sqrt[6]{18}$$

P 3.15

$$\sqrt{\mathrm{a}\sqrt[3]{\mathrm{b}}} = \sqrt{\sqrt[3]{\mathrm{a}^3.\mathrm{b}}} = \sqrt[6]{\mathrm{a}^3.\mathrm{b}}$$

P 3.16

$$\sqrt[5]{\mathrm{a}\sqrt[4]{\mathrm{a}^2\mathrm{b}^3\sqrt[3]{\mathrm{a}^2\mathrm{bc}^{10}}}} = \sqrt[5]{\sqrt[4]{\mathrm{a}^4\mathrm{a}^2\mathrm{b}^3\sqrt[3]{\mathrm{a}^2\mathrm{bc}^{10}}}} = \sqrt[5]{\sqrt[4]{\mathrm{a}^6\mathrm{b}^3\sqrt[3]{\mathrm{a}^2\mathrm{bc}^{10}}}} = \sqrt[20]{\sqrt[3]{\mathrm{a}^{18}\mathrm{b}^9\mathrm{a}^2\mathrm{bc}^{10}}} =$$
$$\sqrt[20]{\sqrt[3]{\mathrm{a}^{20}\mathrm{b}^{10}\mathrm{c}^{10}}} = \sqrt[60]{(\mathrm{a}^2\mathrm{bc})^{10}} = \sqrt[6]{\mathrm{a}^2\mathrm{bc}}$$

P 3.17

$$\sqrt[4]{x^2y^3} : \sqrt[3]{xy} = \sqrt[12]{(x^2y^3)^3} : \sqrt[12]{(xy)^4} = \sqrt[12]{(x^6y^9):(x^4y^4)} = \sqrt[12]{x^2y^5}$$

P 3.18

a)

$$\left(\sqrt{7} - \sqrt{5}\right)^2 = \left(\sqrt{7}\right)^2 - 2.\sqrt{7}.\sqrt{5} + \left(\sqrt{5}\right)^2 = 7 - 2\sqrt{35} + 5 = 12 - 2\sqrt{35}$$

b)

$$\left(2\sqrt{3} + 1\right)\left(2\sqrt{3} - 1\right) = \left(2\sqrt{3}\right)^2 - 1^2 = 2^2.3 - 1 = 12 - 1 = 11$$

c)

Pela regra prática do desenvolvimento do binômio de Newton, o desenvolvimento possui (n+1) termos, isto é, $(a-b)^6$ possui (6+1) = 7 termos:

$$1.a^6b^0 - \frac{1\times6}{(0+1)}a^5b^1 + \frac{6\times5}{(1+1)}a^4b^2 - \frac{15\times4}{(2+1)}a^3b^3 + \frac{20\times3}{(3+1)}a^2b^4 - \frac{15\times2}{(4+1)}a^1b^5 + \frac{6\times1}{(5+1)}a^0b^6$$

Logo, substituindo $a = 2x,\ b = 1$ e efetuando as simplificações, temos:

$$(2x-1)^6 = 64x^6 - 192x^5 + 240x^4 - 160x^3 + 60x^2 - 12x + 1$$

P 3.19

a)

$(a-0{,}3)\left(a+\frac{1}{3}\right)=a^2+\left(-\frac{3}{10}+\frac{1}{3}\right)a+\left(-\frac{3}{10}.\frac{1}{3}\right)=a^2+\left(\frac{-9+10}{30}\right)a+$
$\left(-\frac{1}{10}\right)=a^2+\frac{1}{30}a+-\frac{1}{10}$

b)

Utilizando o produto notável $(a+b)(a^2-a.b+b^2)=a^3+b^3$, temos:

$\left(\frac{4a^2}{9}-\frac{2\sqrt{2}}{3}a+2\right)\left(\frac{2a}{3}+\sqrt{2}\right)=\left(\frac{2}{3}a\right)^3+\left(\sqrt{2}\right)^3=\frac{8}{27}a^3+\sqrt{8}$

c)

Utilizando o produto notável $(a-b)(a^2+a.b+b^2)=a^3-b^3$, temos:

$\left(a-\sqrt{5}\right)\left(a^2+\sqrt{5}a+5\right)=a^3-\left(\sqrt{5}\right)^3=a^3-\sqrt{125}$

P 3.20

Pela fórmula de Bháskara, sabemos que:

$x'=\frac{-b-\sqrt{\Delta}}{2a}$ e $x"=\frac{-b+\sqrt{\Delta}}{2a}$, logo:

$x'+x"=\frac{-b-\sqrt{\Delta}}{2a}+\frac{-b+\sqrt{\Delta}}{2a}=\frac{-b-\sqrt{\Delta}-b+\sqrt{\Delta}}{2a}=\frac{-2b}{2a}=-\frac{b}{a}$

P 3.21

a)

$24b^4d^2-6d^2=6d^2(4b^4-1)=6d^2(2b^2-1)(2b^2+1)$

b)

Utilizando o produto notável $(a-b)^2=a^2-2ab+b^2$, temos:

$7x^5-42x^4y+63x^3y^2=7x^3(x^2-6xy+9y^2)=7x^3(x-3y)^2$

c)

$x^2-2bx^2-5a+10ab=x^2(1-2b)-5a(1-2b)=(x^2-5a)(1-2b)$

d)

Utilizando o produto notável $(a-b)^2=a^2-2ab+b^2$, temos:

$\frac{1}{4}a^2b^4-5ab^3+25b^2=b^2\left(\frac{1}{4}a^2b^2-5ab+25\right)=b^2\left(\frac{1}{2}ab-5\right)^2$

P 3.22

Utilizando o produto notável $(a+b)(a^2 - a.b + b^2) = a^3 + b^3$, temos:

a)

$$\frac{125}{512}a^3 + \frac{8}{27} = \left(\frac{5a}{8} + \frac{2}{3}\right)\left(\frac{25a^2}{64} - \frac{5a}{12} + \frac{4}{9}\right)$$

b)

Utilizando o produto notável $(a-b)(a^2 + a.b + b^2) = a^3 - b^3$, temos:

$a^3 - b^6 = (a - b^2)(a^2 + ab^2 + b^4)$

c)

Utilizando o produto notável $(a-b)(a^2 + a.b + b^2) = a^3 - b^3$, temos:

$a^4 - 125ac^3 = a(a^3 - 125c^3) = a(a - 5c)(a^2 + 5ac + 25c^2)$

P 3.23

a)

$$\frac{x^2-2x+1}{x^2-1} = \frac{(x-1)^2}{(x-1)(x+1)} = \frac{(x-1)}{(x+1)}$$

b)

$$\frac{4(a+1)}{(a^2+2a+1)} : \frac{20(a^2-2a+1)}{a^2-1} = \frac{4(a+1)}{(a+1)^2} : \frac{20(a-1)^2}{(a-1)(a+1)} = \frac{4}{a+1} \times \frac{a+1}{20(a-1)} = \frac{1}{5(a-1)}$$

c)

$$\frac{-x^2+4x-1}{x^2-1} + \frac{x-2}{x+1} = \frac{-x^2+4x-1}{(x-1)(x+1)} + \frac{x-2}{x+1} = \frac{-x^2+4x-1+(x-1)(x-2)}{(x-1)(x+1)} = \frac{-x^2+4x-1+x^2-3x+2}{(x-1)(x+1)} =$$
$$\frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1}$$

d)

Utilizando o produto notável $(a+b)(a^2 - a.b + b^2) = a^3 + b^3$, temos:

$$\frac{27a^3+1}{3a+1} = \frac{(3a+1)(9a^2-3a.1+1^2)}{(3a+1)} = 9a^2 - 3a + 1$$

P 3.24

a)

$$\frac{6\times\sqrt{3}}{\sqrt{3\times\sqrt{3}}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

b)

$$\frac{a\times\sqrt{ab}}{\sqrt{ab\times\sqrt{ab}}} = \frac{a\sqrt{ab}}{ab} = \frac{\sqrt{ab}}{b}$$

c)

$$\frac{2m\times\sqrt[4]{(2m)^3}}{\sqrt[4]{2m}\times\sqrt[4]{(2m)^3}} = \frac{2m\sqrt[4]{8m^3}}{\sqrt[4]{(2m)^4}} = \frac{2m\sqrt[4]{8m^3}}{2m} = \sqrt[4]{8m^3}$$

d)

$$\frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}-1)} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-1^2} = \frac{(\sqrt{3})^2+2.\sqrt{3}.1+1^2}{3-1} = \frac{3+2\sqrt{3}+1}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

P 3.25

a)

$$\frac{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}{(x-3)(\sqrt{x}+\sqrt{3})} = \frac{(\sqrt{x})^2-(\sqrt{3})^2}{(x-3)(\sqrt{x}+\sqrt{3})} = \frac{x-3}{(x-3)(\sqrt{x}+\sqrt{3})} = \frac{1}{\sqrt{x}+\sqrt{3}}$$

b)

$$\frac{(\sqrt{x-2}-1)(\sqrt{x-2}+1)}{(x^2-7x+12)(\sqrt{x-2}+1)} = \frac{(\sqrt{x-2})^2-1^2}{(x-3)(x-4)(\sqrt{x-2}+1)} = \frac{(x-2)-1}{(x-3)(x-4)(\sqrt{x-2}+1)} = \frac{(x-3)}{(x-3)(x-4)(\sqrt{x-2}+1)} = \frac{1}{(x-4)(\sqrt{x-2}+1)}$$

c)

$$\frac{(2-\sqrt{x})(2+\sqrt{x})}{(x^2-16)(2+\sqrt{x})} = \frac{2^2-(\sqrt{x})^2}{(x-4)(x+4)(2+\sqrt{x})} = \frac{4-x}{(x-4)(x+4)(2+\sqrt{x})} = \frac{-1}{(x+4)(2+\sqrt{x})}$$

P 3.26

Utilizando o produto notável $(a-b)(a^2+a.b+b^2)=a^3-b^3$, temos:

a)

$$\frac{x^3-a^3}{x-a}=\frac{(x-a)(x^2+xa+a^2)}{x-a}=x^2+ax+a^2$$

b)

Utilizando o produto notável $(a+b)(a^2-a.b+b^2)=a^3+b^3$, temos:

$$\frac{x^3+1}{x^2+4x+3}=\frac{(x+1)(x^2-x.1+1^2)}{(x+1)(x+3)}=\frac{x^2-x+1}{x+3}$$

P 3.27

Utilizando o produto notável $(a+b)(a-b)=a^2-b^2$, temos:

$$\frac{x^2-25}{x-5}=\frac{(x+5)(x-5)}{x-5}=x+5$$

P 3.28

Utilizando o produto notável $(a-b)(a^2+a.b+b^2)=a^3-b^3$, temos:

$$\frac{2x^2-3x-2}{x^3-8}=\frac{2(x^2-\frac{3}{2}x-1)}{(x-2)(x^2+x.2+2^2)}=\frac{2\left(x+\frac{1}{2}\right)(x-2)}{(x-2)(x^2+2x+4)}=\frac{2x+1}{x^2+2x+4}$$

P 3.29

Utilizando o produto notável $(a+b)(a-b)=a^2-b^2$, temos:

$$\frac{x^2-7x+6}{x^2-1}=\frac{(x-1)(x-6)}{(x+1)(x-1)}=\frac{x-6}{x+1}$$

P 3.30

Utilizando o produto notável $(a+b)(a-b)=a^2-b^2$, temos:

$$\frac{x^2+3x-10}{x^2-4}=\frac{(x-2)(x+5)}{(x+2)(x-2)}=\frac{x+5}{x+2}$$

P 3.31

$\frac{x^4-10x^2+9}{x^3-4x^2+3x}=\frac{x^4-10x^2+9}{x(x^2-4x+3)}$. Façamos $y=x^2$ para achar as raízes do numerador:
$x^4-10x^2+9=y^2-10y+9=0$. Aplicando a fórmula de Bháskara,

$$y=\frac{-(-10)\pm\sqrt{(-10)^2-4.1.9}}{2.1}=\frac{10\pm\sqrt{100-36}}{2}=\begin{cases} y'=\frac{10+\sqrt{64}}{2}=\frac{10+8}{2}=9 \\ y''=\frac{10-\sqrt{64}}{2}=\frac{10-8}{2}=1 \end{cases}$$

Então, temos:
$x^2=9 \Rightarrow x=3$ ou $x=-3$ e $x^2=1 \Rightarrow x=1$ ou $x=-1$
Voltando à expressão:

$$\frac{x^4-10x^2+9}{x^3-4x^2+3x}=\frac{x^4-10x^2+9}{x(x^2-4x+3)}=\frac{(x-3)(x+3)(x-1)(x+1)}{x(x-1)(x-3)}=\frac{(x+3)(x+1)}{x}=\frac{x^2+4x+3}{x}$$

P 3.32

Utilizando o produto notável $(a+b)(a-b)=a^2-b^2$, temos:

$$\frac{x^2-1}{x^2+x-2}=\frac{(x+1)(x-1)}{(x+2)(x-1)}=\frac{x+1}{x+2}$$

P 3.33

$$\frac{3x-6}{x^2-3x+2}=\frac{3(x-2)}{(x-2)(x-1)}=\frac{3}{x-1}$$

P 3.34

$$\frac{x^2+2x-8}{x^2+x-6}=\frac{(x-2)(x+4)}{(x-2)(x+3)}=\frac{x+4}{x+3}$$

P 3.35

Utilizando o produto notável $(a-b)(a^2+a.b+b^2)=a^3-b^3$ e o produto notável $(a+b)(a-b)=a^2-b^2$, temos:

$$\frac{x^3-a^3}{x^2-a^2}=\frac{(x-a)(x^2+x.a+a^2)}{(x+a)(x-a)}=\frac{x^2+ax+a^2}{x+a}$$

P 3.36

$$\frac{x^4+3x^3+x+3}{x^2+4x+3}=\frac{x^3(x+3)+(x+3)}{(x+3)(x+1)}=\frac{(x^3+1)(x+3)}{(x+3)(x+1)}=\frac{x^3+1}{x+1}=\frac{(x+1)(x^2-x.1+1^2)}{x+1}=x^2-x+1$$