Exercícios propostos resolvidos

Capítulo 11 – Conjunto dos Números Complexos

P 11.1

$z_1 + z_2^2 = (2,1) + (1,-3).(1,-3) =$
$= (2,1) + (1-9, -3-3) = (2,1) + (-8,-6) = (-6,-5)$

P 11.2

$(x,y)(x,y) + (1,0) = (0,0)$
$(x^2 - y^2, xy + xy) + (1,0) = (0,0)$
$(x^2 - y^2 + 1,\ 2xy) = (0,0)$
(a) $x^2 - y^2 + 1 = 0$
(b) $2xy = 0 \rightarrow x = 0$ ou $y = 0$
Se $x = 0$, substituindo em (a): $-y^2 + 1 = 0 \rightarrow y = \pm\sqrt{1} \rightarrow y = -1$ ou $y = 1$.
Se $y = 0$, substituindo em (a): $x^2 + 1 = 0 \rightarrow x = \pm\sqrt{-1} \notin \mathbb{R}$.

Resposta: (0, 1) e (0, –1)

P 11.3

a) $z_1 + z_3 = (4,-1) + (-2,1) = (4 + (-2), -1 + 1) = (2,0)$
b) $(z_1 + z_2).z_3 = (4+3, -1+0).(-2,1) =$
$= (7,-1).(-2,1) = (-14 - (-1), 7 + 2) = (-13,9)$

P 11.4

a) (3, 2) = 3 +2i
b) (0, –1) = 0 –1i = –i
c) (x + 2y, x – y) = (x+2y) + (x –y)i

P 11.5

a) Condições: $\begin{cases} x + 4 = 0 \rightarrow x = -4 \\ 9x - 6 \neq 0 \rightarrow x \neq \frac{6}{9} = \frac{2}{3} \end{cases}$
b) Condição: $4x^2 - 9x = 0 \rightarrow x(4x - 9) = 0 \rightarrow x = 0 \vee 4x - 9 = 0 \rightarrow x = \frac{9}{4}$
c) Condições: $\begin{cases} x^2 - 3x + 2 \neq 0 \rightarrow x \neq 1 \wedge x \neq 2 \\ 2x - 1 = 0 \rightarrow x = \frac{1}{2} \end{cases}$

Respostas: a) x = –4 b) x =0 ou x = 9/4 c)x = ½

P 11.6

a) $63 \div 4 = 15$ resto 3, então:
$i^{63} = i^{4\times15+3} = (i^4)^{15} \times i^3 = (1)^{15} \times i^2 \times i = 1 \times (-1) \times i = -i$

b) $78 \div 4 = 19$ resto 2, então:
$(-i)^{78} = (-1)^{78}.(i)^{4\times19+2} = 1 \times (i^4)^{19} \times i^2 = (1)^{19} \times i^2 = 1 \times (-1) = -1$

c) $11 \div 4 = 2$ resto 3, então:
$(-2i)^{11} = (-2)^{11}.(i)^{4\times2+3} = -2048 \times (i^4)^2 \times i^3 = -2048 \times (1)^2 \times i^2 \times i$
$= -2048 \times (-1) \times i = 2048i$

P 11.7

a) (2 +3i) – (5 –2i) = (2–5) + (3–(–2))i = –3 +5i
b) (1/2 +i) – (1/2 – i) = (1/2 –1/2) + (i–(–i)) = 0 +2i = 2i
c) (1/3 + i) + (3/2 –2i) + i – i^2 = (1/3+3/2) + (i +(–2i)+i) –(–1)=
$= \left(\frac{1}{3} + \frac{3}{2} + 1\right) + 0i = \frac{2+9+6}{6} = \frac{17}{6}$

P 11.8

(8 +3i).(2 –2i) = 16 – 16i +6i – $6i^2$ = 16 –10i –6(–1) = 22 – 10i
a) $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i + (-1) = 2i$
b) (7 – 8i).(1 + i) = (7 + 8) + (7– 8)i = 15 – i
c) $(1 - i)^3 = 1^3 - 3.1^2.i + 3.1.i^2 - i^3 = 1 - 3i - 3 + i = -2 - 2i$

Respostas: a) 22 – 10i, b) 2i, c) 15 – i, d) –2 –2i

P 11.9

a) $x + yi = 5 - 3i \rightarrow \begin{cases} x = 5 \\ y = -3 \end{cases}$

b) $x - 15yi = -5i \rightarrow \begin{cases} x = 0 \\ -15y = -5 \rightarrow y = \frac{-5}{-15} = \frac{1}{3} \end{cases}$

c) $(3 + 2i) + 5.(-i) = x + 2yi \rightarrow 3 - 3i = x + 2yi \rightarrow \begin{cases} x = 3 \\ 2y = -3 \rightarrow y = -\frac{3}{2} \end{cases}$

d) $3i(4 - 2i) = x + yi \rightarrow 12i + 6 = x + yi \rightarrow \begin{cases} x = 6 \\ y = 12 \end{cases}$

P 11.10

a) $\frac{z_1}{z_2} = \frac{3+2i}{4-i} \times \frac{4+i}{4+i} = \frac{12+3i+8i-2}{16+1} = \frac{10+11i}{17}$

b) $\frac{3z_1}{2\overline{z_2}} = \frac{3(3+2i)}{2(4+i)} = \frac{9+6i}{8+2i} \times \frac{8-2i}{8-2i} = \frac{72-18i+48i+12}{64+4} = \frac{84+60i}{68} = \frac{21+15i}{17}$

c) $\frac{z_1+z_2}{z_3-z_4} = \frac{(3+2i)+(4-i)}{i-5} = \frac{7+i}{i-5} \times \frac{-i-5}{-i-5} = \frac{-7i-35-i^2-5i}{1+25} = \frac{-34-12i}{26} = \frac{-17-6i}{13}$

d) $\frac{3z_1+\overline{z_1}}{z_3-z_1} = \frac{3(3+2i)+(3-2i)}{i-(3+2i)} = \frac{9+6i+3-2i}{-3-i} = \frac{12+4i}{-3-i} \times \frac{-3+i}{-3+i} = \frac{-36+12i-12i-4}{9+1} = \frac{-40}{10} = -4$

Respostas: a) $\frac{10}{17} + \frac{11}{17}i$, b) $\frac{21}{17} + \frac{15}{17}i$, c) $-\frac{17}{13} - \frac{6}{13}i$, d) -4

P 11.11

a) $\rho = |z| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = 2 =$ módulo de z

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{1}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{\sqrt{3}}{2} \end{array}\right\} \theta = \frac{\pi}{3} =$ argumento de z

a) $\rho = |z| = \sqrt{0^2 + (-3)^2} = \sqrt{0+9} = 3 =$ módulo de z

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{0}{3} = 0 \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{-3}{3} = -1 \end{array}\right\} \theta = \frac{3\pi}{2} =$ argumento de z

b) $\rho = |z| = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \sqrt{3+1} = 2 =$ módulo de z

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{\sqrt{3}}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{-1}{2} \end{array}\right\} \theta = \frac{11\pi}{6} =$ argumento de z

P 11.12

a) $\rho = |z| = \sqrt{(6)^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$

b) $\rho = |z| = \sqrt{(-3)^2 + \left(\sqrt{7}\right)^2} = \sqrt{9+7} = \sqrt{16} = 4$

c) $\rho = |z| = \sqrt{(0)^2 + (-9)^2} = \sqrt{0+81} = 9$

P 11.13

a) $\rho = |z| = \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = \sqrt{3+1} = 2$

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{\sqrt{3}}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{1}{2} \end{array}\right\} \theta = \frac{\pi}{6} \rightarrow z_1 = 2\left(\cos\left(\frac{\pi}{6}\right) + \text{ i sen }\left(\frac{\pi}{6}\right)\right)$

a) $\rho = |z| = \sqrt{(1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{1}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{-\sqrt{3}}{2} \end{array}\right\} \theta = \frac{5\pi}{3} \rightarrow z_2 = 2\left(\cos\left(\frac{5\pi}{3}\right) + \text{ i sen }\left(\frac{5\pi}{3}\right)\right)$

b) $\rho = |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{1/2}{1} = \frac{1}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2} \end{array}\right\} \theta = \frac{\pi}{3} \rightarrow z_3 = 1\left(\cos\left(\frac{\pi}{3}\right) + \text{ i sen }\left(\frac{\pi}{3}\right)\right)$

c) $\rho = |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

$\left.\begin{array}{l} \cos\theta = \frac{a}{\rho} = \frac{-1/2}{1} = -\frac{1}{2} \\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2} \end{array}\right\} \theta = \frac{2\pi}{3} \rightarrow z_4 = 1\left(\cos\left(\frac{2\pi}{3}\right) + \text{ i sen }\left(\frac{2\pi}{3}\right)\right)$

Respostas: a) $z_1 = 2\left(\cos\left(\frac{\pi}{6}\right) + \text{ i sen }\left(\frac{\pi}{6}\right)\right)$

b) $z_2 = 2\left(\cos\left(\frac{5\pi}{3}\right) + \text{ i sen }\left(\frac{5\pi}{3}\right)\right)$

c) $z_3 = 1\left(\cos\left(\frac{\pi}{3}\right) + \text{ i sen }\left(\frac{\pi}{3}\right)\right)$

d) $z_4 = 1\left(\cos\left(\frac{2\pi}{3}\right) + \text{ i sen }\left(\frac{2\pi}{3}\right)\right)$

P 11.14

a) Se z = (a, b), então –z = (–a, –b), ou seja, está no 3º quadrante. Nesse caso, $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

b) Se z = (a, b), então $\bar{z}$ = (a, –b), ou seja, está no 4º quadrante. Nesse caso, $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

c) Se z = (a, b), então $-\bar{z}$ = (–a, b), ou seja, está no 2º quadrante. Nesse caso, $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

P 11.15

$$z_1.z_2 = 2\sqrt{2}\left(\cos\left(\frac{3\pi}{8}+\frac{11\pi}{8}\right)+ \text{ i sen }\left(\frac{3\pi}{8}+\frac{11\pi}{8}\right)\right)$$
$$= 2\sqrt{2}\left(\cos\left(\frac{14\pi}{8}\right)+ \text{ i sen }\left(\frac{14\pi}{8}\right)\right) = 2\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+ \text{ i sen }\left(\frac{7\pi}{4}\right)\right)$$
$$= 2\sqrt{2}\left(\cos\left(\pi+\frac{\pi}{4}\right)+ \text{ i sen }\left(\pi+\frac{\pi}{4}\right)\right) = 2\sqrt{2}\left(\frac{\sqrt{2}}{2}+\text{i}\left(-\frac{\sqrt{2}}{2}\right)\right)$$
$$= 2 - 2\text{i}$$

P 11.16

$$z_1.z_2 = 3.1\left(\cos\left(\frac{7\pi}{10}+\frac{\pi}{5}\right)+ \text{ i sen }\left(\frac{7\pi}{10}+\frac{\pi}{5}\right)\right)$$
$$= 3\left(\cos\left(\frac{7\pi+2\pi}{10}\right)+ \text{ i sen }\left(\frac{7\pi+2\pi}{10}\right)\right)$$
$$= 3\left(\cos\left(\frac{9\pi}{10}\right)+ \text{ i sen }\left(\frac{9\pi}{10}\right)\right)$$

P 11.17

$$z^6 = 2^6\left(\cos\left(6\times\frac{\pi}{3}\right)+ \text{ i sen }\left(6\times\frac{\pi}{3}\right)\right) = 64(\cos(2\pi)+ \text{ i sen }(2\pi)) = 64$$

P 11.18

$$\rho = |z| = \sqrt{(1)^2+\left(\sqrt{3}\right)^2} = \sqrt{1+3} = 2$$
$$\left.\begin{array}{l}\cos\theta = \frac{a}{\rho} = \frac{1}{2}\\ \text{sen }\theta = \frac{b}{\rho} = \frac{\sqrt{3}}{2}\end{array}\right\} \theta = \frac{\pi}{3} \rightarrow z = 2\left(\cos\left(\frac{\pi}{3}\right)+ \text{ i sen }\left(\frac{\pi}{3}\right)\right)$$
$$z^3 = 2^3\left(\cos\left(3\times\frac{\pi}{3}\right)+ \text{ i sen }\left(3\times\frac{\pi}{3}\right)\right) = 8(\cos(\pi)+ \text{ i sen }(\pi)) = -8$$

P 11.19

$$z = 2\left(\cos\left(\frac{\pi}{8}\right)+ \text{ i sen }\left(\frac{\pi}{8}\right)\right) \rightarrow z^2 = 2^2\left(\cos\left(2\times\frac{\pi}{8}\right)+ \text{ i sen }\left(2\times\frac{\pi}{8}\right)\right) =$$
$$= 4\left(\cos\left(\frac{\pi}{4}\right)+ \text{ i sen }\left(\frac{\pi}{4}\right)\right) = 4\left(\frac{\sqrt{2}}{2}+\text{i}\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}+2\sqrt{2}i$$

P 11.20

$$\rho = |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$$

$$\left.\begin{aligned} \cos\theta &= \frac{a}{\rho} = \frac{1/2}{\sqrt{2}/2} = \frac{\sqrt{2}}{2} \\ \operatorname{sen}\theta &= \frac{b}{\rho} = \frac{1/2}{\sqrt{2}/2} = \frac{\sqrt{2}}{2} \end{aligned}\right\} \theta = \frac{\pi}{4} \;\rightarrow z = \frac{\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4}\right) + \text{ i sen }\left(\frac{\pi}{4}\right)\right)$$

$$z^{12} = \left(\frac{\sqrt{2}}{2}\right)^{12}\left(\cos\left(12\times\frac{\pi}{4}\right) + \text{ i sen }\left(12\times\frac{\pi}{4}\right)\right) = \frac{1}{64}(\cos(3\pi) + \text{ i sen }(3\pi))$$

$$= -\frac{1}{64}$$

P 11.21

$$\rho = |z| = \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$$

$$\left.\begin{aligned} \cos\theta &= \frac{a}{\rho} = \frac{-4}{4} = -1 \\ \operatorname{sen}\theta &= \frac{b}{\rho} = \frac{0}{4} = 0 \end{aligned}\right\} \theta = \pi \;\rightarrow z = 4(\cos(\pi) + \text{ i sen }(\pi))$$

$$z^{1/4} = \sqrt[4]{4}\left(\cos\left(\frac{\pi+2k\pi}{4}\right) + \text{ i sen }\left(\frac{\pi+2k\pi}{4}\right)\right)$$

$$= \sqrt{2}\left(\cos\left(\frac{\pi+2k\pi}{4}\right) + \text{ i sen }\left(\frac{\pi+2k\pi}{4}\right)\right), \text{para k} = 0{,}1, \text{e } 3.$$

$$z_0 = \sqrt{2}\left(cos\left(\frac{\pi}{4}\right) + isen\left(\frac{\pi}{4}\right)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + \text{i}\frac{\sqrt{2}}{2}\right) = 1 + \text{i}$$

$$z_1 = \sqrt{2}\left(cos\left(\frac{3\pi}{4}\right) + isen\left(\frac{3\pi}{4}\right)\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + \text{i}\frac{\sqrt{2}}{2}\right) = -1 + \text{i}$$

$$z_2 = \sqrt{2}\left(cos\left(\frac{5\pi}{4}\right) + isen\left(\frac{5\pi}{4}\right)\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - \text{i}\frac{\sqrt{2}}{2}\right) = -1 - \text{i}$$

$$z_3 = \sqrt{2}\left(cos\left(\frac{7\pi}{4}\right) + isen\left(\frac{7\pi}{4}\right)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} - \text{i}\frac{\sqrt{2}}{2}\right) = 1 - \text{i}$$

Resposta: S = {1 + i; –1 + i; –1 – i; 1 – i}

P 11.22

Seja $w = a + bi$, de tal maneira que $w^2 = z$, ou seja:

$$(a + bi)^2 = 5 - 12i \quad \rightarrow \quad (a^2 - b^2) + 2abi = 5 - 12i$$

Logo,

$$\begin{cases} a^2 - b^2 = 5 \\ 2ab = -12 \end{cases} \rightarrow a = -\frac{6}{b} \;\rightarrow\; \left(-\frac{6}{b}\right)^2 - b^2 = 5 \;\rightarrow\; \frac{36}{b^2} - b^2 = 5$$

$36 - b^4 = 5b^2 \;\rightarrow\; b^4 + 5b^2 - 36 = 0$, é uma equação biquadrada. Vamos fazer uma substituição $t = b^2$:

$$t^2 + 5t - 36 = 0 \;\rightarrow\; t = \frac{-5 \pm \sqrt{25 - 4.1.(-36)}}{2.1} = \frac{-5 \pm 13}{2} = \begin{cases} t_1 = 4 \\ t_2 = -9 \end{cases}$$

Para $t_1 = 4,\; b^2 = 4,$ ou seja b $= \pm 2$. Para b = 2, temos $a = -\frac{6}{2} = -3$ e para b $= - 2$, temos $a = -\frac{6}{-2} = 3$.

Resposta: S = {3 – 2i; –3 + 2i}

P 11.23

Vamos escrever $z_k = -2$ na forma polar:

ρ = |z| = $\sqrt{(-2)^2 + (0)^2} = \sqrt{4} = 2$

$$\left.\begin{aligned} \cos\theta &= \frac{a}{\rho} = \frac{-2}{2} = -1 \\ \text{sen}\,\theta &= \frac{b}{\rho} = 2 = 0 \end{aligned}\right\} \theta = \pi \;\rightarrow z_k = 2(\cos(\pi) + \text{ i sen }(\pi))$$

Pela segunda fórmula de De Moivre, temos:

$$u_k = \sqrt[6]{r}\left[\cos\left(\frac{\theta + 2k\pi}{6}\right) + \text{isen}\left(\frac{\theta + 2k\pi}{6}\right)\right]$$

para k = 0, 1, 2, 3, 4 e 5.

Então,

$$\begin{cases} \sqrt[6]{r} = 2 \;\rightarrow r = 2^6 = 64 \\ \dfrac{\theta + 2k\pi}{6} = \pi, \text{ para algum k inteiro.} \end{cases}$$

Neste caso, $\theta + 2k\pi = 6\pi$, para valores de k inteiros.

Se tomarmos, por exemplo, k = 3, então:

$$u_k = 2\left[\cos\left(\frac{0 + 2k\pi}{6}\right) + \text{isen}\left(\frac{0 + 2k\pi}{6}\right)\right]$$

para k = 0, 1, 2, 3, 4 e 5.

$z_0 = 2\big(cos(0) + isen(0)\big) = 2$

$z_1 = 2\left(cos\left(\frac{\pi}{3}\right) + isen\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} + \text{i}\frac{\sqrt{3}}{2}\right) = 1 + \sqrt{3}\text{i}$

$z_2 = 2\left(cos\left(\frac{2\pi}{3}\right) + isen\left(\frac{2\pi}{3}\right)\right) = 2\left(-\frac{1}{2} + \text{i}\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}\text{i}$

$z_3 = 2\big(cos(\pi) + isen(\pi)\big) = 2(-1 + \text{i}.0) = -2$

$z_4 = 2\left(cos\left(\frac{4\pi}{3}\right) + isen\left(\frac{4\pi}{3}\right)\right) = 2\left(-\frac{1}{2} - \text{i}\frac{\sqrt{3}}{2}\right) = -1 - \sqrt{3}\text{i}$

$z_5 = 2\left(cos\left(\frac{5\pi}{3}\right) + isen\left(\frac{5\pi}{3}\right)\right) = 2\left(\frac{1}{2} - \text{i}\frac{\sqrt{3}}{2}\right) = 1 - \sqrt{3}\text{i}$

Resposta: S = {2; $-1-\sqrt{3}i$; $1-\sqrt{3}i$; 2; $1+\sqrt{3}i$; $-1+\sqrt{3}i$}

P 11.24

ρ = |z| = $\sqrt{(64)^2+(0)^2} = 64$

$$\left.\begin{aligned}\cos\theta = \frac{a}{\rho} = \frac{64}{64} = 1\\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{0}{64} = 0\end{aligned}\right\}\ \theta = 0 \ \rightarrow z = 64(\cos(0) + \text{ i sen }(0))$$

$$z^{1/4} = \sqrt[4]{64}\left(\cos\left(\frac{0+2k\pi}{4}\right) + \text{ i sen }\left(\frac{0+2k\pi}{4}\right)\right)$$

$$= 2\sqrt{2}\left(\cos\left(\frac{2k\pi}{4}\right) + \text{ i sen }\left(\frac{2k\pi}{4}\right)\right), \text{para } k = 0,1, 2 \text{ e } 3.$$

$$z_0 = 2\sqrt{2}\big(cos(0) + isen(0)\big) = 2\sqrt{2}(1 + \mathrm{i}0) = 2\sqrt{2}$$

$$z_1 = 2\sqrt{2}\left(cos\left(\frac{\pi}{2}\right) + isen\left(\frac{\pi}{2}\right)\right) = 2\sqrt{2}(0 + \mathrm{i}.1) = 2\sqrt{2}\ \mathrm{i}$$

$$z_2 = 2\sqrt{2}\big(cos(\pi) + isen(\pi)\big) = 2\sqrt{2}(-1 - \mathrm{i}.0) = -2\sqrt{2}$$

$$z_3 = 2\sqrt{2}\left(cos\left(\frac{3\pi}{2}\right) + isen\left(\frac{3\pi}{2}\right)\right) = 2\sqrt{2}(0 - \mathrm{i}.1) = -2\sqrt{2}\ \mathrm{i}$$

Resposta: S = {$2\sqrt{2}$; $2\sqrt{2}$i; $-2\sqrt{2}$; $-2\sqrt{2}$i}

P 11.25

a) $3x^4 - 48 = 0 \Leftrightarrow x^4 = \frac{48}{3} \Leftrightarrow x^4 = 16$

Escrevendo z = 16, na forma trigonométrica,

$|z| = \rho = \sqrt{(16)^2 + 0^2} = 16$

$$\left.\begin{aligned}\cos\theta = \frac{a}{\rho} = \frac{16}{16} = 1\\ \text{sen}\,\theta = \frac{b}{\rho} = \frac{0}{16} = 0\end{aligned}\right\}\ \theta = 0 \ \Rightarrow z = 16(\cos 0 + \text{ i sen } 0)$$

Aplicando a segunda fórmula de De Moivre, temos:

$$u_k = \sqrt[4]{16}\left[\cos\left(\frac{0+2k\pi}{4}\right) + \text{isen}\left(\frac{0+2k\pi}{4}\right)\right]$$

para k = 0, 1, 2 e 3.

$u_0 = 2[\cos 0 + \text{isen}0] = 2$

$u_1 = 2\left[\cos\left(\frac{\pi}{2}\right) + \text{isen}\left(\frac{\pi}{2}\right)\right] = 2i$

$u_2 = 2[\cos(\pi) + \text{isen}(\pi)] = -2$

$u_3 = 2\left[\cos\left(\frac{3\pi}{2}\right) + \text{isen}\left(\frac{3\pi}{2}\right)\right] = -2i$

Resposta: S ={–2; 2; –2i; 2i}

b) $x^3 + 2i = 0 \Leftrightarrow x^3 = -2i$

Escrevendo z = $-2i$, na forma trigonométrica,

$|z| = \rho = \sqrt{0^2 + (-2)^2} = 2$

$$\left.\begin{aligned} \cos\theta &= \frac{a}{\rho} = \frac{0}{2} = 0 \\ \text{sen}\,\theta &= \frac{b}{\rho} = \frac{-2}{2} = -1 \end{aligned}\right\} \theta = \frac{3\pi}{2} \quad \Rightarrow z = 2\left(\cos\frac{3\pi}{2} + \text{ i sen } \frac{3\pi}{2}\right)$$

Aplicando a segunda fórmula de De Moivre, temos:

$$u_k = \sqrt[3]{2}\left[\cos\left(\frac{3\pi/2 + 2k\pi}{3}\right) + \text{isen}\left(\frac{3\pi/2 + 2k\pi}{3}\right)\right]$$

para k = 0, 1 e 2.

$u_0 = \sqrt[3]{2}\left[\cos\frac{\pi}{2} + \text{isen}\frac{\pi}{2}\right] = \sqrt[3]{2}i$

$u_1 = \sqrt[3]{2}\left[\cos\left(\frac{7\pi}{6}\right) + \text{isen}\left(\frac{7\pi}{6}\right)\right] = \sqrt[3]{2}(\frac{-\sqrt{3}}{2} - \frac{1}{2}i)$

$u_2 = \sqrt[3]{2}\left[\cos\left(\frac{11\pi}{6}\right) + \text{isen}\left(\frac{11\pi}{6}\right)\right] = \sqrt[3]{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i)$

Resposta: S = { $\sqrt[3]{2}i$; $\sqrt[3]{2}(\frac{-\sqrt{3}}{2} - \frac{1}{2}i)$; $\sqrt[3]{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i)$}

P 11.26

a) Aplicando a fórmula de Bháskara para determinar o valor de z, temos:

$$z = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 53}}{2 \times 1} = \frac{4 \pm \sqrt{-196}}{2} = \frac{4 \pm 14\text{i}}{2} = 2 \pm 7\text{i}$$

b) Aplicando a fórmula de Bháskara para determinar o valor de x, temos:

$$z = \frac{(2+2i) \pm \sqrt{(2+2i)^2 - 4 \times i \times (2-i)}}{2 \times i} = \frac{(2+2i) \pm \sqrt{4 + 8i + 4i^2 - 8i + 4i^2}}{2i}$$

$$= \frac{(2+2i) \pm \sqrt{4 - 4 - 4}}{2i} = \frac{2 + 2i \pm 2i}{2i}$$

$$= \begin{cases} \dfrac{2+4i}{2i} \times \dfrac{i}{i} = \dfrac{2i - 4}{-2} = 2 - i \\ \dfrac{2}{2i} \times \dfrac{i}{i} = \dfrac{2i}{-2} = -i \end{cases}$$

Respostas: a) S ={2 +7i; 2 – 7i}   b) S = {– i, 2 – i}

P 11.27

$$2z^2 - 3z + 2 = 2.\left(\frac{3}{4} + \frac{\sqrt{7}}{4}i\right)^2 - 3.\left(\frac{3}{4} + \frac{\sqrt{7}}{4}i\right) + 2$$

$$= 2.\left(\frac{(9-7) + 6\sqrt{7}i}{16}\right) - \frac{9 + 3\sqrt{7}}{4} + 2 = \frac{2 + 6\sqrt{7}}{8} - \frac{9 + 3\sqrt{7}}{4} + 2$$

$$= 2\left(\frac{1 + 3\sqrt{7}}{8}\right) - \frac{9 + 3\sqrt{7}}{4} + 2 = \frac{1 + 3\sqrt{7} - 9 - 3\sqrt{7}}{4} + 2 = -\frac{8}{4} + 2$$

$$= 0$$

Logo, satisfaz a equação